Colorization by Matrix Completion

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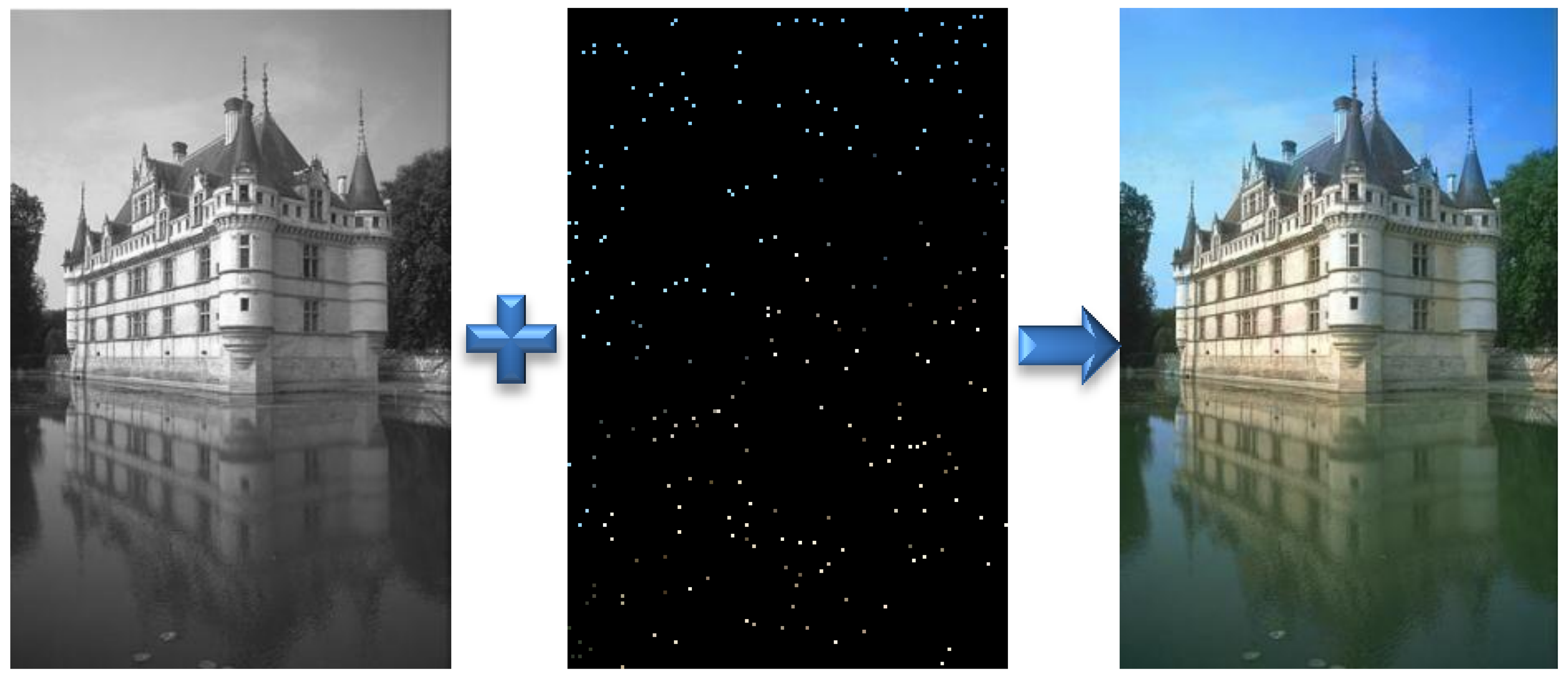
Problem Definition

Suppose we are given a monochrome image $\mathbf{W} \in \mathbb{R}^{m\times n}$, a partially observed color image $\mathbf{D} \in \mathbb{R}^{m\times 3n}$, and a zero-one matrix $\boldsymbol{\Omega} \in \{0,1\}^{m\times 3n}$, where $\Omega_{ij} = 1$ indicates D_{ij} is observed and $\Omega_{ij} = 0$ otherwise. The colorization problem is to obtain the three color components $\mathbf{R}, \mathbf{G}, \mathbf{B} \in \mathbb{R}^{m\times n}$ which best approximate the underlying $\tilde{\mathbf{R}}, \tilde{\mathbf{G}}, \tilde{\mathbf{B}}$, respectively.



Monochrome **Labels** **Colorized**

Model

Suppose the underlying color image is the sum of a low-rank component $\mathbf{L}$ and a sparse component $\mathbf{S}$, then the colorization problem can be solved by the following optimization problem.

$$\min_{\mathbf{R},\mathbf{G},\mathbf{B},\mathbf{L},\mathbf{S}} \quad \|\mathbf{L}\|_* + \lambda \|\boldsymbol{\Omega} \circ \mathbf{S}\|_1;$$

$$\text{s.t.} \quad \mathbf{L} = [\mathbf{R}, \mathbf{G}, \mathbf{B}];$$

$$\mathbf{L} + \mathbf{S} = \mathbf{D};$$

$$\alpha_1 \mathbf{R} + \alpha_2 \mathbf{G} + \alpha_3 \mathbf{B} = \mathbf{W},$$

This is actually a variant of the matrix completion model of [1].

References

[1] E. Candès, X. Li, Y. Ma, and J. Wright: *Robust Principal Component Analysis*. Journal of the ACM 58(3), 2011.

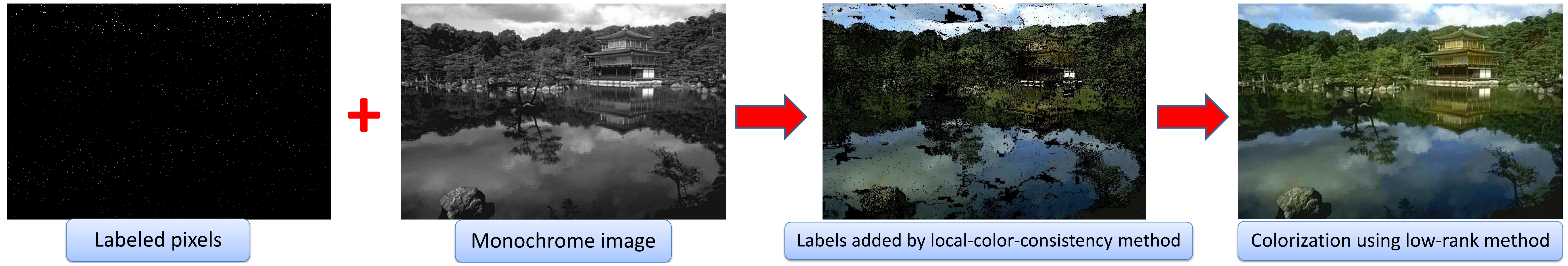
[2] A. Levin, D. Lischinski, and Y. Weiss. *Colorization Using Optimization*. ACM Transactions on Graphics (TOG), 2004.

Further Improvement

Advantage: robust to noises and incorrect labels

Disadvantage: have poor performance when very few pixels are labeled (say less than 10%)

Improvement: first generate more labels (probably incorrect), then incorporate the initial labels and generated labels for colorization. (How to generate labels? Neighboring pixels have similar intensity tend to have similar color.)



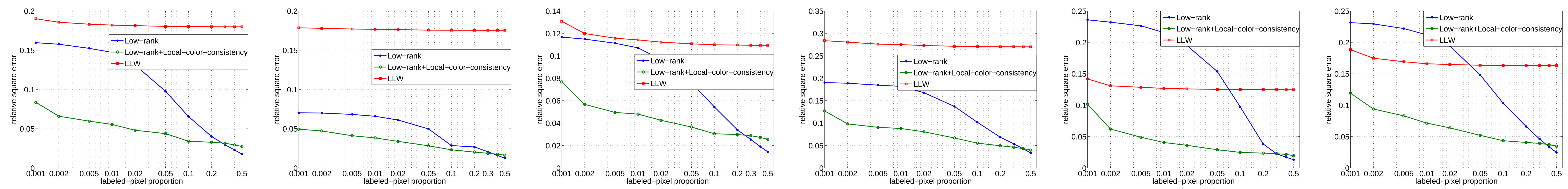
Experiments

Compared methods:

- *low rank*: solution to our low-rank matrix completion model (without generating labels)
- *low rank + local color consistency*: solution to our model (generating more labels by exploiting local color consistency)
- *LLW*: the method of Levin, Lischinski, and Weiss, 2004, [2].

Relative Square Error: $RSE = \frac{\|\mathbf{L}^* - [\tilde{\mathbf{R}}, \tilde{\mathbf{G}}, \tilde{\mathbf{B}}]\|_F}{\|[\tilde{\mathbf{R}}, \tilde{\mathbf{G}}, \tilde{\mathbf{B}}]\|_F}$

We randomly hold a proportion of pixels (uniformly) of some color images as the labels, then run each method and report the RSE.



Original **Monochrome** **Labels** **LLW** **low rank** **low rank + local color consistency**